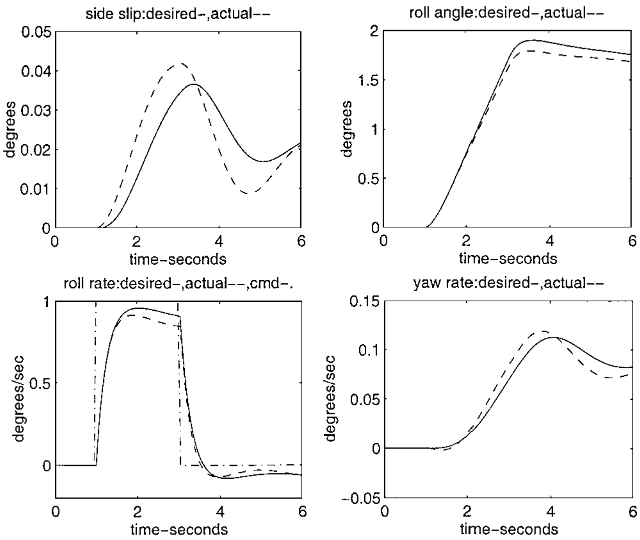

Fig. 2 Time history responses: roll-rate command.

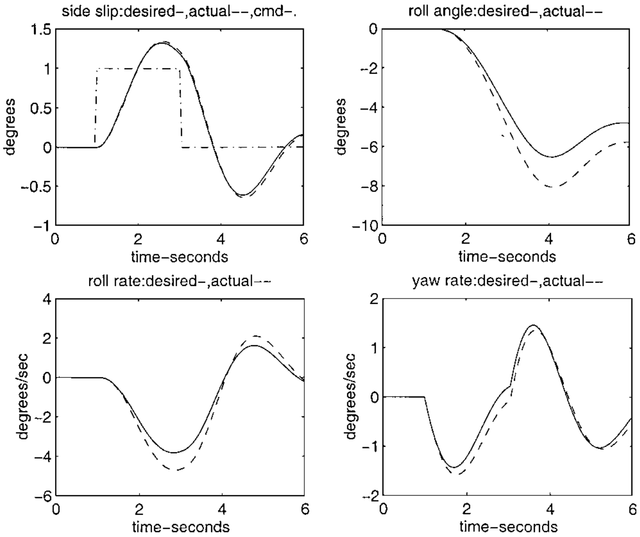

Fig. 3 Time history responses: side slip command.

with a natural frequency of 2 rad/s. There is coupling between side slip and roll rate as specified in the model for desired response.

Time history responses to a 2-s step, roll-rate command are shown in Fig. 2. It can be seen that the desired and actual responses provide first-order tracking of the command and that the directional response is very small. Figure 3 shows time history responses to a 2-s step, side-slip command. A second-order response is achieved and a positive side slip results in a negative roll response as is required for static lateral stability.

Because the aircraft is unstable, robustness of the controller is important. It was found that the size of the B matrix could be reduced by 18% before the aircraft became unstable.

Conclusions

The method outlined in this Note provides a simple model for generating desired eigenvalues and eigenvectors for use in eigenstructure design of lateral-directional controllers for aircraft.

Acknowledgment

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Transfer Function Parameter Changes Due to Structural Damage

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Introduction

THE damage detection of large flexible structures is an important and challenging task in the system identification and structural dynamics communities. Recently, various damage detection approaches have been proposed.[1–3] Most of them use the finite element model (FEM) update techniques. In general, the FEM update techniques require many sensors to measure mode shapes, but the number of sensors is limited in practical applications. Recently, a correlation approach[1] was developed for damage detection based on the comparison of the transfer function parameter change of the tested system and the change due to damage. Only a few sensors are required for this approach. The results of a comparison of this method with some other methods show that this method is more robust to model inaccuracy such as the error due to noise than the other methods.[4]

This Note presents a novel study of the transfer function parameter change due to structural damage by applying this correlation approach. This extends the correlation approach to the study of the characteristics of the parameter change due to structural damage such as the reduction of element stiffness. The characteristics of the parameter change play an important role in pattern recognition for methods such as neural network techniques, which can be applied to structural damage detection. Because only a few sensors are required for the correlation approach, the placement of sensors is important. This Note also addresses the issue of optimal sensor placement for structural damage detection. The presented sensor placement technique is based on the sensitivity of the transfer function parameter to structural damage.

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Correlation Approach

A brief introduction to the correlation approach[1] is given in this section. To concisely present this approach, we consider a flexible structure with single input and m displacement outputs. Here we use the first k-mode analytical model, which may be an FEM. The transfer functions of the healthy structure are

$$g_j(s) = \sum_{i=1}^{k} \frac{b_{ji}}{s^2 + a_{1i}s + a_{0i}}, \qquad j = 1, 2, \ldots, m \tag{1}$$

The corresponding parameter vectors of the healthy structure are defined as

$$\begin{aligned} \boldsymbol{p}_0 &= [a_{01} \quad a_{11} \quad a_{02} \quad a_{12} \quad \cdots \quad a_{0k} \quad a_{1k}]^T \\ \boldsymbol{p}_j &= [b_{j1} \quad b_{j2} \quad \cdots \quad b_{jk}]^T, \qquad j = 1, 2, \ldots, m \end{aligned} \tag{2}$$

The changes of the parameter vectors of the ith damage case are defined as

$$\Delta\boldsymbol{p}_{ij} = \boldsymbol{p}_{ij} - \boldsymbol{p}_j, \qquad j = 0, 1, \ldots, m \tag{3}$$

where $\boldsymbol{p}_{ij}$ are the parameter vectors of the ith damage case. Because of the perturbation of each parameter, we define the weighted change vectors as

$$\Delta\boldsymbol{p}_{ij}^W = \left[\frac{\Delta\boldsymbol{p}_{ij}(1)}{W_{j1}}, \quad \ldots, \quad \frac{\Delta\boldsymbol{p}_{ij}(k)}{W_{jk}}\right]^T \tag{4}$$

where $\Delta\boldsymbol{p}_{ij}(l)$ is the lth element of $\Delta\boldsymbol{p}_{ij}$ and W_{jl} is the standard deviation of $\{\Delta\boldsymbol{p}_{1j}(l), \Delta\boldsymbol{p}_{2j}(l), \ldots, \Delta\boldsymbol{p}_{nj}(l)\}$ for the considered n damage cases. The correlations between the tested system with the weighted change vectors $\Delta\boldsymbol{p}_j^W$, which represent the difference between the tested system and the healthy system, and the ith damage case are defined as

$$C_{ij} = \frac{\left(\Delta\boldsymbol{p}_j^W\right)^T \Delta\boldsymbol{p}_{ij}^W}{\left|\Delta\boldsymbol{p}_j^W\right|\left|\Delta\boldsymbol{p}_{ij}^W\right|}, \qquad j = 0, 1, \ldots, m \tag{5}$$

The correlation C_{ij} represents the cosine between two vectors. The value of the correlation C_{ij} is between -1 and 1. When C_{ij} is less than 0, the change vector $\Delta\boldsymbol{p}_j^W$ of the tested system is in a different direction (>90 deg) from the change vector $\Delta\boldsymbol{p}_{ij}^W$ due to the ith element damage. It strongly implies that the ith element is not damaged. The minimum correlation of the tested system corresponding to the ith damage case is defined as

$$C_i = \min\{C_{i0}, C_{i1}, \ldots, C_{im}\} \tag{6}$$

The magnitude ratios between the tested system and the ith damage case are defined as

$$R_{ij} = \frac{\left|\Delta\boldsymbol{p}_j^W\right|}{\left|\Delta\boldsymbol{p}_{ij}^W\right|}, \qquad j = 0, 1, \ldots, m \tag{7}$$

In the real application, the vectors $\Delta\boldsymbol{p}_j^W$ are obtained from the identified parameters of the tested system and the healthy system, and the referred vectors $\Delta\boldsymbol{p}_{ij}^W$ are based on analytical models such as FEM.

Sensor Placement

The study of the characteristics of the parameter change due to damage is based on the following equation:

$$\Delta\boldsymbol{p}_j^W(\tau_1, \ldots, \tau_i + \Delta\tau_i, \ldots, \tau_n) \approx f_{ij}(\Delta\tau_i)\left(\delta\boldsymbol{p}_{ij}^W\right) \tag{8}$$

where

$$\delta\boldsymbol{p}_{ij}^W = \left.\frac{\mathrm{d}\Delta\boldsymbol{p}_j^W}{\mathrm{d}\Delta\tau_i}\right|_{\Delta\tau_i = 0} \tag{9}$$

where variable $\Delta\tau_i$ is the damage parameter related to the ith element and $\Delta\boldsymbol{p}_j^W$ is the weighted change of the jth parameter vector $\boldsymbol{p}_j$. Here the value of the variable τ_i, which corresponds to the healthy structure, is 1. A negative value of $\Delta\tau_i$ represents the level of damage. For example, $\Delta\tau_i = -0.2$ means that the damage of the ith element is 20%. The vectors $\delta\boldsymbol{p}_{ij}^W$ represent the parametric sensitivity to the ith element damage. The right term in Eq. (8) represents the projection of $\Delta\boldsymbol{p}_j^W$ in the direction of $\delta\boldsymbol{p}_{ij}^W$.

Because only a few sensors are necessary for the correlation approach, sensor placement is very important. A technique is presented that determines the placement of sensors based on the change of the transfer function parameter due to damage. The cost function of the sensor placement for the lth mode is defined as

$$S_{lj} = \sum_{i=1}^{n} \left|\delta\boldsymbol{p}_{ij}^W(l)\right|, \qquad j = 1, \ldots, N \tag{10}$$

where $\delta\boldsymbol{p}_{ij}^W(l)$ is the sensitivity of the lth mode at node j, which is the jth possible sensor location. The variable S_{lj} represents the summation of the absolute value of the parametric sensitivity to each damage for the lth mode at the jth node. Each mode has one associated cost function S_{lj}. In reality, some identified modes are more accurate and important than others. For the considered k modes, we define the weighted normalized cost function as

$$S_j = \sum_{l=1}^{k} W_l S_{lj}^0 \tag{11}$$

where

$$S_{lj}^0 = S_{lj}/X_l, \qquad X_l = \max\{S_{l1}, S_{l2}, \ldots, S_{lN}\} \tag{12}$$

where S_{lj}^0 is the normalized cost function for the lth mode and W_l is the weighting factor for the lth mode.

Results and Discussion

The FEM of a cantilevered aluminum Euler beam,[5] as shown in Fig. 1, is used for study. The length, width, and thickness of this beam are 1, 0.038, and 0.0016 m, respectively. This Note presents the results based on the first three modes with natural frequencies, 1.30, 8.20, and 23.9 Hz, respectively. We consider the damage due to stiffness loss. First, sensor placement is discussed. An actuator located at node 5 is used to excite the vibration in the y direction. For the sensor placement, each mode is considered equally, so that the weights W_l are 1. Figure 2 shows the results of cost function S_j. For these three modes, the cost function S_7 is close to the maximum cost function S_{15}. Two sensors located at nodes 7 and 15 are used.

Next, we examine the results for element 8 with stiffness losses of 10, 30, and 50%. Here the referred vectors $\Delta\boldsymbol{p}_{ij}^W$ used to compute

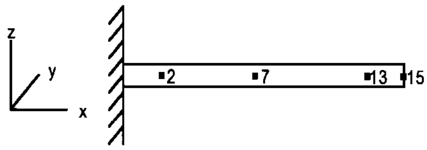


Fig. 1 Cantilevered Euler beam.

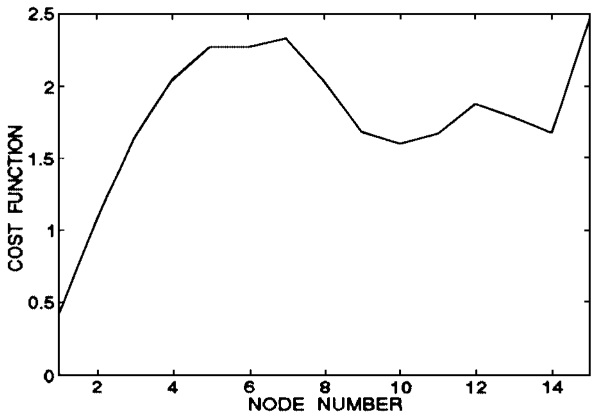


Fig. 2 Cost function S_j for sensor placement.

Table 1 Minimum correlation and magnitude ratio

Stiffness loss	10%	30%	50%
C_8	1.0000	0.9998	0.9988
C_7	0.7991	0.8041	0.8127
R_{8j}			
$j=0$	1.0000	3.6789	7.9266
$j=1$	1.0000	3.7600	8.4149
$j=2$	1.0000	3.7117	8.1260
R_{7j}			
$j=0$	1.1170	4.1094	8.8542
$j=1$	0.4978	1.8717	4.1889
$j=2$	1.6165	5.9999	13.127

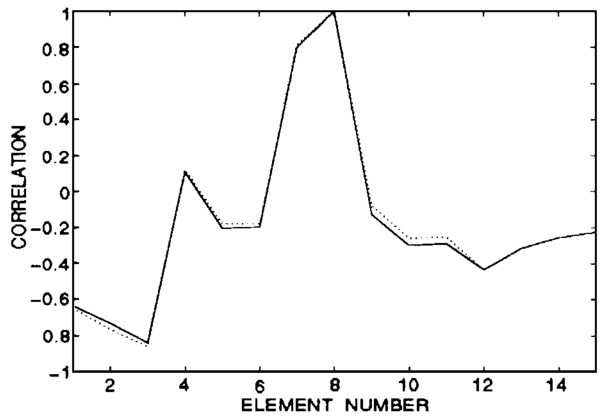


Fig. 3 Minimum correlation C_i for element 8 with stiffness loss: ——, 10%, and ·····, 50%.

correlation C_{ij} and magnitude ratios are the weighted parameter changes due to 10% stiffness loss of the ith element. Figure 3 shows that the minimum correlation can distinguish the damage element with various stiffness losses by using the referred systems with 10% stiffness loss. The minimum correlation C_8 for the damage element 8 has negligible change when stiffness loss changes from 10 to 50%. This implies that the direction of $\Delta \boldsymbol{p}_j^W$ has negligible change for various stiffness losses.

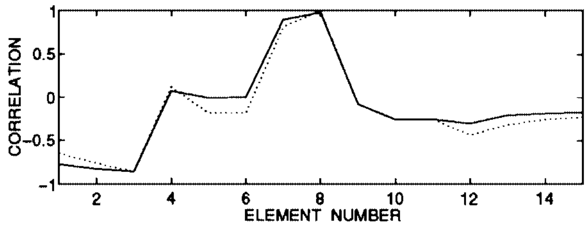
Table 1 lists the minimum correlations and magnitude ratios for elements 7 and 8 when element 8 has various stiffness losses. The minimum correlation C_8 has negligible change when stiffness loss varies from 10 to 50%. This verifies that the right term in Eq. (8) is almost the same as the left term in Eq. (8) with negligible error. For each stiffness loss case (10, 30, and 50%), the three magnitude ratios R_{8j} $(j=0, 1, 2)$ are close to each other. This implies that f_{8j} $(j=0, 1, 2)$ in Eq. (8) are close to each other. The minimum correlation C_7 is also high and close to 1. But the magnitude ratios R_{7j} $(j=0, 1, 2)$ are not close for each stiffness loss case. The magnitude ratios can distinguish the damage element from the other elements with high minimum correlation, and they also indicate the intensity of damage.[5]

The effect of sensor placement is examined by comparing two different options of sensor placement. The first option is the preceding one with two sensors located at nodes 7 and 15, whereas the second one has two sensors located at nodes 2 and 13. The results from the application of the OKID[6] algorithm to analyze time-domain response data are used for comparison. The OKID algorithm can identify the system model with negligible error from noise-free data. The simulated time-domain response data are obtained from the state-space model

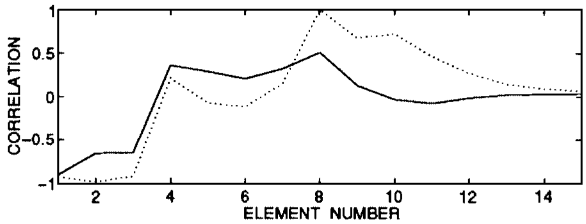
$$x(k+1) = Ax(k) + Bu(k) + v(k) \tag{13}$$

$$z(k) = Cx(k) + w(k) \tag{14}$$

where $[A \;\; B \;\; C]$ is the discrete state-space model, x is the state vector, u is the input, z is the output, v is the process noise, and w is the measurement noise. The input u and noise v and w are chosen as Gaussian distributed random data with zero mean. We examine the results for element 8 with 50% stiffness loss.



Option 1



Option 2

Fig. 4 Minimum correlation C_i with various sensor options: ——, noise, and ·····, noise free.

Figure 4 shows the minimum correlation of the noise-free and noise cases. The results in Fig. 4 are obtained by applying Eq. (5) to the vectors $\Delta \boldsymbol{p}_j^W$ based on the identified models and the vectors $\Delta \boldsymbol{p}_{ij}^W$ based on FEMs. For the noise-free case, both options can identify the damage position. For the noise case, option 1 with optimal sensor placements can identify the damage position, but option 2 cannot identify the damage position. Option 2 is much more sensitive to noise than option 1. This indicates that the use of more sensors may not be good for this correlation approach. Also, some other results[5] show that noise dramatically affects the magnitude ratios for option 2, and noise has much less effect on the magnitude ratios for option 1.

Conclusions

A novel study of the characteristics of the transfer function parameter change due to structural damage is presented. The results show that the direction of the weighted change vector $\Delta \boldsymbol{p}_j^W$ has negligible change for various levels of damage. This extends the application of the correlation approach. The magnitude ratios can be used to distinguish the damage element when many minimum correlations are close to 1, and they can also be used to indicate the intensity of damage. A sensor placement technique based on the sensitivity of the weighted parameter change to damage is also presented. The results show that the damage detection with sensors at the optimal positions is less sensitive to noise.

Acknowledgments

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